**1.1**

**2. Which one of these are propositions? What are the truth values of those that are propositions?**

1. **Do not pass go.**

Not a declarative sentence, so *not a proposition*

1. **What time is it?**

Not a declarative sentence*, not a proposition*

1. **There are no black flies in Maine.**

Declarative sentence, this is a proposition. The truth value is F

1. **4 + x = 5**

Neither True nor False, *not a proposition*

1. **The moon is made of green cheese.**

Declarative Sentence, proposition. The truth value is F

1. **2n ≥ 100.**

Neither true nor false, *not a proposition*

**14**. **Let p, q, and r be the propositions**

**p: You get an A on the final exam**

**q: You do every exercise in this book**

**r: You get an A in this class**

**Write these propositions using p,q, and r and logical connectives (including negations)**

1. **You get an A in this class, but you do not do every exercise in this book**

r ^ ⌐q

1. **You get an A on the final, you do every exercise in this book, and you get an A in this class.**

p ^ q ^ r

1. **To get an A in this class, it is necessary for you to get an A on the final.**

r → p

1. **You get an A on the final, but you don’t do every exercise in this book; nevertheless, you get an A in this class.**

(P ^ ⌐ q) ^ r

1. **Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.**

(p ^ q) → r

1. **You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.**

(p ᵛ q) ↔ r

**24. Write each of these statements in the form “if *p*, then *q*” in English. [*Hint:* Refer to the list of common ways to express conditional statements provided in this section.]**

1. **I will remember to send you the address only if you send me an e-mail message.**

If I remember to send you the address then you have sent me an email message

1. **To be a citizen of this country, it is sufficient that you were born in the United States.**

If you were born in the United States then you are a citizen of this country

1. **If you keep your textbook, it will be a useful reference in your future courses.**

If you keep your text book then you’ll have a useful reference in your future courses

1. **The Red Wings will win the Stanley Cup if their goalie plays well.**

If the Red Wings goalie plays well then the Red Wings will win the Stanley Cup

1. **That you get the job implies that you had the best credentials.**

If you get the job then you had the best credentials

1. **The beach erodes whenever there is a storm.**

If there is a storm, then the beach erodes

1. **It is necessary to have a valid password to log on to the server.**

If you can log on to the server then you have a valid password

1. **You will reach the summit unless you begin your climb too late.**

If you don’t start your climb too late then you will reach the summit

**32. Construct a truth table for each of these compound propositions.**

**a) *p* →￢*p***

|  |  |  |
| --- | --- | --- |
| **P** | **⌐p** | **p→⌐p** |
| **T** | **F** | **F** |
| **F** | **T** | **T** |

**b) *p* ↔￢*p***

|  |  |  |
| --- | --- | --- |
| **p** | **⌐p** | **p↔⌐p** |
| **T** | **F** | **F** |
| **F** | **T** | **F** |

**c) *p* ⊕ *(p* ∨ *q)***

|  |  |  |  |
| --- | --- | --- | --- |
| **p** | **q** | **p V q** | **p ⊕ (p ∨ q)** |
| **T** | **T** | **T** | **F** |
| **T** | **F** | **T** | **F** |
| **F** | **T** | **T** | **T** |
| **F** | **F** | **F** | **F** |

**d) *(p* ∧ *q)* → *(p* ∨ *q)***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **p ^ q** | **p V q** | **(p ∧ q) → (p ∨ q)** |
| **T** | **T** | **T** | **T** | **T** |
| **T** | **F** | **F** | **T** | **T** |
| **F** | **T** | **F** | **T** | **T** |
| **F** | **F** | **F** | **F** | **T** |

**e) *(q* →￢*p)* ↔ *(p* ↔ *q)***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **p** | **q** | **⌐p** | **q →￢p** | **p ↔ q** | **(q →￢p) ↔ (p ↔ q)** |
| **T** | **T** | **F** | **F** | **T** | **F** |
| **T** | **F** | **F** | **T** | **F** | **F** |
| **F** | **T** | **T** | **T** | **F** | **F** |
| **F** | **F** | **T** | **T** | **T** | **T** |

**f) *(p* ↔ *q)* ⊕ *(p* ↔￢*q)***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **P** | **q** | **⌐q** | **p ↔ q** | **p↔⌐q** | ***(p* ↔ *q)* ⊕ *(p* ↔￢*q)*** |
| **T** | **T** | **F** | **T** | **F** | **T** |
| **T** | **F** | **T** | **F** | **T** | **T** |
| **F** | **T** | **F** | **F** | **T** | **T** |
| **F** | **F** | **T** | **T** | **F** | **T** |

**36. Construct a truth table for each of these compound propositions.**

**a) *(p* ∨ *q)* ∨ *r***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p V q** | **(p V q) V r** |
| **T** | **T** | **T** | **T** | **T** |
| **T** | **T** | **F** | **T** | **T** |
| **T** | **F** | **T** | **T** | **T** |
| **T** | **F** | **F** | **T** | **T** |
| **F** | **T** | **T** | **T** | **T** |
| **F** | **T** | **F** | **T** | **T** |
| **F** | **F** | **T** | **F** | **T** |
| **F** | **F** | **F** | **F** | **F** |

**b) *(p* ∨ *q)* ∧ *r***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p V q** | **(p V q) ^ r** |
| **T** | **T** | **T** | **T** | **T** |
| **T** | **T** | **F** | **T** | **F** |
| **T** | **F** | **T** | **T** | **T** |
| **T** | **F** | **F** | **T** | **F** |
| **F** | **T** | **T** | **T** | **T** |
| **F** | **T** | **F** | **T** | **F** |
| **F** | **F** | **T** | **F** | **F** |
| **F** | **F** | **F** | **F** | **F** |

**c) *(p* ∧ *q)* ∨ *r***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p ^ q** | **(p ^ q) V r** |
| **T** | **T** | **T** | **T** | **T** |
| **T** | **T** | **F** | **T** | **T** |
| **T** | **F** | **T** | **F** | **T** |
| **T** | **F** | **F** | **F** | **F** |
| **F** | **T** | **T** | **F** | **T** |
| **F** | **T** | **F** | **F** | **F** |
| **F** | **F** | **T** | **F** | **T** |
| **F** | **F** | **F** | **F** | **F** |

**d) *(p* ∧ *q)* ∧ *r***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p ^ q** | **(p ^ q) ^ r** |
| **T** | **T** | **T** | **T** | **T** |
| **T** | **T** | **F** | **T** | **F** |
| **T** | **F** | **T** | **F** | **F** |
| **T** | **F** | **F** | **F** | **F** |
| **F** | **T** | **T** | **F** | **F** |
| **F** | **T** | **F** | **F** | **F** |
| **F** | **F** | **T** | **F** | **F** |
| **F** | **F** | **F** | **F** | **F** |

**e) *(p* ∨ *q)*∧￢*r***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **P** | **q** | **r** | **⌐r** | **p V q** | **(p V q) ^ ⌐r** |
| **T** | **T** | **T** | **F** | **T** | **F** |
| **T** | **T** | **F** | **T** | **T** | **T** |
| **T** | **F** | **T** | **F** | **T** | **F** |
| **T** | **F** | **F** | **T** | **T** | **T** |
| **F** | **T** | **T** | **F** | **T** | **F** |
| **F** | **T** | **F** | **T** | **T** | **T** |
| **F** | **F** | **T** | **F** | **F** | **F** |
| **F** | **F** | **F** | **T** | **F** | **F** |

**f) *(p* ∧ *q)*∨￢*r***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **P** | **q** | **r** | **⌐r** | **p ^ q** | **(p ^ q) V ⌐r** |
| **T** | **T** | **T** | **F** | **T** | **T** |
| **T** | **T** | **F** | **T** | **T** | **T** |
| **T** | **F** | **T** | **F** | **F** | **F** |
| **T** | **F** | **F** | **T** | **F** | **T** |
| **F** | **T** | **T** | **F** | **F** | **F** |
| **F** | **T** | **F** | **T** | **F** | **T** |
| **F** | **F** | **T** | **F** | **F** | **F** |
| **F** | **F** | **F** | **T** | **F** | **T** |

**1.3**

**8. Use De Morgan’s laws to find the negation of each of the following statements.**

**⌐ (p ᴧ q) ≡ ⌐p v ⌐q**

**⌐ (p v q) ≡ ⌐p ᴧ ⌐q**

1. **Kwame will take a job in industry or go to graduate school.**

p = Kwame will take a job in industry

q = Kwame will go to graduate school

p v q

Negation = ⌐ (p v q) is equivalent to ⌐p ᴧ ⌐q

Kwame will not take a job in industry and he will not go to graduate school

1. **Yoshiko knows Java and calculus.**

p = Yoshiko knows Java

q = Yoshiko knows Calculus

p ᴧ q

Negation ⌐(p ᴧ q) which is equivalent to ⌐p v ⌐q

Yoshiko does not know Java or he does not know Calculus

1. **James is young and strong.**

p = James is young

q = James is strong

p ᴧ q

Negation ⌐ (p ᴧ q) which is equivalent to ⌐p ᴧ ⌐q

James is not young or he is not strong

1. **Rita will move to Oregon or Washington.**

p = Rita will move to Oregon

q = Rita will move to Washington

Negation ⌐ (p v q) is equivalent to ⌐p ᴧ ⌐q

Rita will not move to Oregon and she will not move to Washington

**14. Determine whether (⌐p ∧ (p → q)) → ⌐q is a tautology.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **⌐p** | **(p → q)** | **(⌐p ∧ (p → q))** | **⌐q** | **(⌐p ∧ (p → q)) → ⌐q** |
| **T** | **T** | **F** | **T** | **F** | **F** | **T** |
| **T** | **F** | **F** | **F** | **F** | **T** | **T** |
| **F** | **T** | **T** | **T** | **T** | **F** | **F** |
| **F** | **F** | **T** | **T** | **T** | **T** | **T** |

p → q ≡ ⌐p v q Example 3

(⌐p ∧ (p → q)) → ⌐q ≡ (⌐p ∧ (⌐p v q)) → ⌐q

(⌐p ∧ (⌐p v q)) → ⌐q ≡ (⌐p ∧ ⌐p) v (⌐p ∧ q) → ⌐q Distributive Law

(⌐p ∧ ⌐p) v (⌐p ∧ q) → ⌐q ≡ ⌐ (p v p) v (⌐p ∧ q) → ⌐q De Morgan’s First Law

⌐ (p v p) v (⌐p ∧ q) → ⌐q ≡ ⌐ p v (⌐p ∧ q) → ⌐q Idempotent Laws

⌐ p v (⌐p ∧ q) → ⌐q ≡ ⌐ p → ⌐q Absorption Law

⌐ p → ⌐q ≡ ⌐ (⌐ p) v ⌐q Logical Equivalences Involving Conditional Statements

⌐ (⌐ p) v ⌐q ≡ p v ⌐q Double Negation Law

**p v ⌐q cannot be simplified further meaning it is a contingency**

**16. Show that *p* ↔ *q* and *(p* ∧ *q)* ∨ *(*￢*p* ∧￢*q)* are logically equivalent**

|  |  |  |
| --- | --- | --- |
| p | q | p ↔ q |
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| p | q | p ^ q | ⌐p | ⌐q | ￢p ∧￢q | (p ∧ q) ∨ (￢p ∧￢q) |
| T | T | T | F | F | F | T |
| T | F | F | F | T | F | F |
| F | T | F | T | F | F | F |
| F | F | F | T | T | T | T |

**20. Show that ￢ *(p* ⊕ *q)* and *p* ↔ *q* are logically equivalent.**

|  |  |  |
| --- | --- | --- |
| p | q | p ↔ q |
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

|  |  |  |  |
| --- | --- | --- | --- |
| p | q | (p ⊕ q) | ￢ (p ⊕ q) |
| T | T | F | T |
| T | F | T | F |
| F | T | T | F |
| F | F | F | T |

**32. Show that *(p* ∧ *q)* → *r* and *(p* → *r)* ∧ *(q* → *r)* are not logically equivalent.**

( p ^ q) → r ≡ ⌐(p ^ q) V r Example 3

⌐ (p ^ q) V r ≡ r V ⌐ (p ^ q) Commutative Laws

r V ⌐ (p ^ q) ≡ r V (⌐p V ⌐q) De Morgan’s 1st Law

(p → r) ∧ (q → r) ≡ (⌐p V r) ^ (⌐q V r) Example 3

(⌐p V r) ^ (⌐q V r) ≡ r V (⌐p ^ ⌐q)

(⌐p ^ ⌐q) does not equal (⌐p V ⌐q) so they are not logically equivalent